Math 1510 Week 9

Trig Substitution

eg $\displaystyle\int \frac{dx}{\sqrt{9-x^2}}$

Sol Let $x = 3\sin\theta \quad dx = 3\cos\theta\, d\theta$

$$\int \frac{dx}{\sqrt{9-x^2}} = \int \frac{3\cos\theta\, d\theta}{\sqrt{9-9\sin^2\theta}}$$

$$= \int \frac{3\cos\theta\, d\theta}{\sqrt{9\cos^2\theta}}$$

$$= \int \frac{3\cos\theta\, d\theta}{3\cos\theta} \quad (*)$$

$$= \int d\theta$$

$$= \theta + C$$

$$= \arcsin\frac{x}{3} + C$$

Rmk $(*)$ For substitution $x = 3\sin\theta$

we take $\theta = \arcsin\frac{x}{3} \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

and so $\cos\theta \geq 0 \Rightarrow \sqrt{\cos^2\theta} = \cos\theta$

$$\boxed{\begin{array}{l} \text{Generally, if } a>0 \text{ is a constant} \\ \text{For } \sqrt{a^2-x^2} \text{ try } x = a\sin\theta \\ \text{For } \sqrt{x^2-a^2} \text{ try } x = a\sec\theta \\ \text{For } \sqrt{a^2+x^2} \text{ try } x = a\tan\theta \end{array}}$$

Remember

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

eg $\displaystyle\int \frac{dx}{x^2\sqrt{x^2-4}}$, $x > 2$

$\sec\theta > 1$

$\boxed{\text{Can take } \theta \in (0, \frac{\pi}{2})}$

Let $x = 2\sec\theta$, then $dx = 2\sec\theta\tan\theta\, d\theta$

$$\int \frac{dx}{x^2\sqrt{x^2-4}} = \int \frac{2\sec\theta\tan\theta\, d\theta}{4\sec^2\theta\sqrt{4\sec^2\theta-4}}$$

$$= \frac{1}{2}\int \frac{\tan\theta\, d\theta}{\sec\theta\sqrt{4\tan^2\theta}}$$

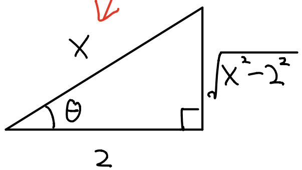
$\tan\theta > 0$

$$= \frac{1}{2}\int \frac{\cos\theta\tan\theta\, d\theta}{2\tan\theta}$$

$$= \frac{1}{4}\int \cos\theta\, d\theta$$

$$= \frac{1}{4}\sin\theta + C$$

Can find $\sin\theta$ by drawing triangle

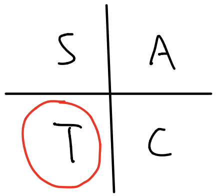


$$= \frac{\sqrt{x^2-4}}{4x} + C$$

Rmk What if $x<-2$ in last example?

In that case $x = 2\sec\theta < 0 \Rightarrow \cos\theta < 0$

We can take θ in quadrant III $\left(\theta \in [\pi, \frac{3\pi}{2})\right)$



Then $\tan\theta \geq 0$

$$\Rightarrow \sqrt{\tan^2\theta} = \tan\theta$$

Also, $\sin\theta \leq 0$

$$\Rightarrow \sin\theta = -\sqrt{1-\cos^2\theta}$$

$$= -\sqrt{1 - \frac{1}{\sec^2\theta}}$$

$$= -\sqrt{1 - \frac{2^2}{x^2}}$$

$$= -\sqrt{\frac{x^2-4}{x^2}}$$

$$= \frac{\sqrt{x^2-4}}{x}$$

$\left(\begin{array}{l} x<0 \\ \Rightarrow \sqrt{x^2} = -x \end{array} \right)$

eg
$$\int \frac{dx}{\sqrt{x^2+2x+3}}$$

$$= \int \frac{dx}{\sqrt{(x+1)^2+2}}$$

$\left(\begin{array}{l} \text{Let } x+1 = \sqrt{2}\tan\theta \\ dx = \sqrt{2}\sec^2\theta\, d\theta \end{array} \right)$

$$= \int \frac{\sqrt{2}\sec^2\theta\, d\theta}{\sqrt{2\tan^2\theta+2}}$$

$$= \int \frac{\sqrt{2}\sec^2\theta\, d\theta}{\sqrt{2\sec^2\theta}}$$

$$\overset{(1)}{=} \int \frac{\sqrt{2}\sec^2\theta\, d\theta}{\sqrt{2}\sec\theta}$$

$$= \int \sec\theta\, d\theta$$

$$= \ln\left|\sec\theta + \tan\theta\right| + C$$

$$\overset{(2)}{=} \ln\left|\frac{\sqrt{x^2+2x+3}+x+1}{\sqrt{2}}\right| + C$$

If $\theta = \arctan\frac{x+1}{\sqrt{2}}$

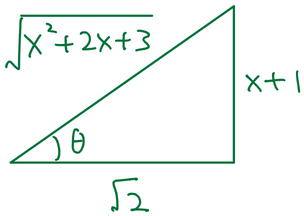
then $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\Rightarrow \sec\theta > 0$

$\therefore \sqrt{\sec^2\theta} = \sec\theta$ (1)

$\Rightarrow \sec\theta = \sqrt{1+\tan^2\theta}$

(2) $= \sqrt{1+\frac{(x+1)^2}{2}}$



t-substitution (or t-formula)

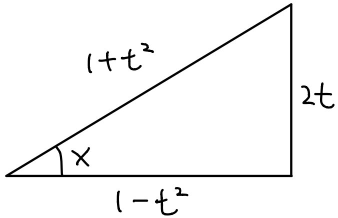
Let $t = \tan\frac{x}{2}$. Then

$$\tan x = \frac{2t}{1-t^2} \qquad dx = \frac{2}{1+t^2}\,dt$$

$$\sin x = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

Picture $(0 < x < \frac{\pi}{2})$



$$\begin{aligned}&(1-t^2)^2 + (2t)^2\\ &= 1 - 2t^2 + t^4 + 4t^2\\ &= 1 + 2t^2 + t^4\\ &= (1+t)^2\end{aligned}$$

Pf $\quad \dfrac{2t}{1-t^2} = \dfrac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} = \tan x$

$$\frac{2t}{1+t^2} = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{2\tan\frac{x}{2}}{\sec^2\frac{x}{2}} = 2\sin\frac{x}{2}\cos\frac{x}{2} = \sin x$$

$$\frac{1-t^2}{1+t^2} = \frac{1-t^2}{2t}\cdot\frac{2t}{1+t^2} = \frac{1}{\tan x}\cdot\sin x = \cos x$$

$$t = \tan\frac{x}{2} \Rightarrow \frac{dt}{dx} = \frac{1}{2}\sec^2\frac{x}{2} = \frac{1}{2}(1+t^2) \Rightarrow dx = \frac{2}{1+t^2}\,dt$$

t-substitution is useful for integrating rational functions in $\sin x, \cos x$

i.e. $\dfrac{\text{polynomial in } \sin x, \cos x}{\text{polynomial in } \sin x, \cos x}$

eg $\displaystyle\int \csc x\,dx$

$$\begin{aligned}&= \int \frac{1}{\sin x}\,dx \qquad \left(\text{Let } t = \tan\frac{x}{2}\right)\\ &= \int \frac{1+t^2}{2t}\,\frac{2\,dt}{1+t^2}\\ &= \int \frac{dt}{t}\\ &= \ln|t| + C\\ &= \ln\left|\tan\frac{x}{2}\right| + C\end{aligned}$$

Rmk $\quad \ln\left|\tan\frac{x}{2}\right| = -\ln|\csc x + \cot x|$

eg $\displaystyle\int \frac{1}{1-\cos x}\,dx$ (Let $t=\tan\frac{x}{2}$)

$$= \int \frac{1}{1-\frac{1-t^2}{1+t^2}} \cdot \frac{2}{1+t^2}\,dt$$

$$= \int \frac{2\,dt}{1+t^2-(1-t^2)}$$

$$= \int \frac{dt}{t^2}$$

$$= -\frac{1}{t} + C$$

$$= -\cot\frac{x}{2} + C$$

eg $\displaystyle\int \frac{dx}{4\sin x + 3\cos x + 3}$ (Let $t=\tan\frac{x}{2}$)

$$= \int \frac{1}{4\left(\frac{2t}{1+t^2}\right)+3\left(\frac{1-t^2}{1+t^2}\right)+3} \cdot \frac{2\,dt}{1+t^2}$$

$$= \int \frac{2\,dt}{8t+3-3t^2+3+3t^2}$$

$$= \int \frac{dt}{4t+3}$$

$$= \frac{1}{4}\int \frac{d(4t+3)}{4t+3}$$

$$= \frac{1}{4}\ln|4t+3| + C$$

$$= \frac{1}{4}\ln\left|4\tan\frac{x}{2}+3\right| + C$$

Integration of rational functions

Recall: By long division and partial fractions

Rational function = Polynomial + Partial fractions

(Integer Analogue: $\frac{49}{10} = 4 + \frac{9}{10}$
$= 4 + \frac{1}{2} + \frac{2}{5}$)

eg

$$\frac{4x^6+x^4-1}{x^4-1} = 4x^2+1+\frac{4x^2}{x^4-1}$$
$$= 4x^2+1+\frac{1}{x-1}-\frac{1}{x+1}+\frac{2}{x^2+1}$$

$$\Rightarrow \int \frac{4x^6+x^4-1}{x^4-1}\,dx = \frac{4}{3}x^3+x+\ln|x-1| - \ln|x+1| + 2\arctan x + C$$

Integrate polynomial : Easy

Integrate partial fractions: Standard

Terms appear in partial fractions:

- $\frac{A}{ax+b}$ or $\frac{A}{(ax+b)^k}$, $k>1$

Easy to integrate : $dx = \frac{1}{a}d(ax+b)$

- $\frac{Ax+B}{ax^2+bx+c}$ or $\frac{Ax+B}{(ax^2+bx+c)^k}$, $k>1$, $\Delta = b^2-4ac<0$

Trig. substitution, Completing square & Reduction formula

Useful formula

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\frac{x}{a}+C \quad (*)$$

$$\int \tan\theta\, d\theta = \ln|\sec\theta| + C$$

Pf of (*)

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\int \frac{1}{1+\left(\frac{x}{a}\right)^2}\, d\left(\frac{x}{a}\right) = \frac{1}{a}\arctan\frac{x}{a}+C$$

$$\boxed{\int \frac{A}{(ax+b)^k}\,dx\ ,\quad k \geq 1}$$

eg $\displaystyle\int \frac{3}{2x+1}\,dx$

$$= \frac{3}{2}\int \frac{d(2x+1)}{2x+1}$$

$$= \frac{3}{2}\ln|2x+1| + C$$

eg $\displaystyle\int \frac{3}{(2x+1)^4}\,dx$

$$= \frac{3}{2}\int \frac{d(2x+1)}{(2x+1)^4}$$

$$= \frac{3}{2}\left(-\frac{1}{3}\right)\frac{1}{(2x+1)^3} + C$$

$$= -\frac{1}{2(2x+1)^3} + C$$

$$\boxed{\int \frac{A}{(ax^2+bx+c)^k}\,dx,\quad k \geq 1,\quad ax^2+bx+c \text{ irreducible}}$$

eg $\displaystyle\int \frac{1}{(x^2+1)^2}\,dx$

let $x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$$= \int \frac{\sec^2\theta\, d\theta}{(\tan^2\theta + 1)^2}$$

$$= \int \frac{\sec^2\theta\, d\theta}{\sec^4\theta}$$

$$= \int \cos^2\theta\, d\theta$$

$$= \frac{1}{2}\int (1+\cos 2\theta)\, d\theta$$

$$= \frac{1}{2}\theta + \frac{1}{4}\int \cos 2\theta\, d2\theta$$

$$= \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta + C$$

$$= \frac{1}{2}\theta + \frac{1}{4}\cdot\frac{2x}{1+x^2} + C$$

See t-formula

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$= \frac{1}{2}\arctan x + \frac{x}{2(1+x^2)} + C$$

$$\int \frac{Ax+B}{ax^2+bx+c}\,dx, \quad ax^2+bx+c \text{ irreducible}$$

eg $$\int \frac{4x+7}{x^2+2x+5}\,dx$$

Method I : Split numerator into 2 parts.

$$d(x^2+2x+5) = (2x+2)\,dx$$

$$4x+7 = 2(2x+2)+3$$

$$\int \frac{4x+7}{x^2+2x+5}\,dx$$

$$= \int \frac{2(2x+2)\,dx}{x^2+2x+5} + \int \frac{3\,dx}{x^2+2x+5}$$

$$= 2\int \frac{d(x^2+2x+5)}{x^2+2x+5} + \int \frac{3\,d(x+1)}{(x+1)^2+2^2}$$

$$= 2\ln(x^2+2x+5) + \frac{3}{2}\arctan\frac{x+1}{2} + C$$

Method II : Trig substitution

$$x^2+2x+5 = (x+1)^2+2^2$$

Let $x+1 = 2\tan\theta$, $\quad x = 2\tan\theta - 1$

$$dx = 2\sec^2\theta\,d\theta$$

$$\int \frac{4x+7}{x^2+2x+5}\,dx = \int \frac{8\tan\theta-4+7}{(2\tan\theta)^2+2^2}\cdot 2\sec^2\theta\,d\theta$$

$$= \int \frac{8\tan\theta+3}{4\sec^2\theta}\,2\sec^2\theta\,d\theta$$

$$= \frac{1}{2}\int (8\tan\theta+3)\,d\theta$$

$$= 4\ln|\sec\theta| + \frac{3}{2}\theta + C$$

$$= 4\ln\sqrt{1+\left(\frac{x+1}{2}\right)^2} + \frac{3}{2}\arctan\frac{x+1}{2} + C$$

$$= 4\ln\sqrt{\frac{x^2+2x+5}{4}} + \frac{3}{2}\arctan\frac{x+1}{2} + C$$

Both correct ?

Sure ! $$4\ln\sqrt{\frac{x^2+2x+5}{4}} = 2\left(\ln(x^2+2x+5) - \ln 4\right) = 2\ln(x^2+2x+5) - 2\ln 4$$

eg $\int \sec x \, dx$

Method I: t-formula

Let $t = \tan\frac{x}{2}$

$$\int \sec x \, dx = \int \frac{dx}{\cos x}$$
$$= \int \frac{1+t^2}{1-t^2} \frac{2dt}{1+t^2}$$
$$= \int \frac{2dt}{1-t^2}$$
$$= \int \left(\frac{1}{t-1} - \frac{1}{t+1}\right) dt$$
$$= \ln|t-1| - \ln|t+1| + C$$
$$= \ln\left|\frac{t-1}{t+1}\right| + C$$
$$= \ln\left|\frac{\tan\frac{x}{2}-1}{\tan\frac{x}{2}+1}\right| + C$$

Method II:

$$\int \sec x \, dx = \int \frac{dx}{\cos x}$$
$$= \int \frac{\cos x \, dx}{\cos^2 x}$$

Let $u = \sin x$

$du = \cos x \, dx$

$$= \int \frac{du}{1-u^2}$$
$$= \frac{1}{2} \int \left(\frac{1}{u+1} - \frac{1}{u-1}\right) du$$
$$= \frac{1}{2}\left(\ln|u+1| - \ln|u-1|\right) + C$$
$$= \frac{1}{2} \ln\left|\frac{\sin x + 1}{\sin x - 1}\right| + C$$

Rmk $\frac{1}{2}\ln\left|\frac{\sin x+1}{\sin x-1}\right| = \frac{1}{2}\ln\left|\frac{(\sin x+1)^2}{(\sin x+1)(\sin x-1)}\right|$

$$= \frac{1}{2}\ln\left|\frac{(\sin x+1)^2}{\sin^2 x-1}\right|$$
$$= \ln\left|\frac{\sin x+1}{\cos x}\right|$$
$$= \ln|\tan x + \sec x|$$

Rationalization

eg $\displaystyle\int \frac{\sqrt{x}}{x+1}\,dx$

Sol Let $u=\sqrt{x}=x^{\frac{1}{2}}$

$\dfrac{du}{dx}=\dfrac{1}{2}x^{-\frac{1}{2}}=\dfrac{1}{2\sqrt{x}}=\dfrac{1}{2u}$

$\therefore\ dx=2u\,du$

$$\int\frac{\sqrt{x}}{x+1}\,dx=\int\frac{u}{u^2+1}(2u\,du)$$

Long division

$$=2\int\frac{u^2\,du}{u^2+1}$$

$$=2\int\frac{u^2+1-1}{u^2+1}\,du$$

$$=2\int\left(1-\frac{1}{u^2+1}\right)du$$

$$=2u-2\arctan u+C$$

$$=2\sqrt{x}-2\arctan\sqrt{x}+C$$

eg $\displaystyle\int\frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx$

Let $u=x^{\frac{1}{6}}$

$x=u^6 \quad dx=6u^5\,du$

$\sqrt{x}=x^{\frac{1}{2}}=u^3$

$\sqrt[3]{x}=x^{\frac{1}{3}}=u^2$

$$=\int\frac{u^3}{1+u^2}\cdot 6u^5\,du$$

$$=6\int\frac{u^8}{1+u^2}\,du$$

long division

$$=6\int\left(u^6-u^4+u^2-1+\frac{1}{1+u^2}\right)du$$

$$=6\left(\frac{u^7}{7}-\frac{u^5}{5}+\frac{u^3}{3}-u+\arctan u\right)+C$$

$$=\frac{6}{7}x^{\frac{7}{6}}-\frac{6}{5}x^{\frac{5}{6}}+2x^{\frac{1}{2}}-6x^{\frac{1}{3}}+\arctan x^{\frac{1}{6}}+C$$

Ex $\displaystyle\int\frac{\sqrt[3]{x}}{1+\sqrt{x}}\,dx$ (Rationalization, long division, partial fraction, trig substitution)

Ans $\displaystyle\frac{6}{5}x^{\frac{5}{6}}-3x^{\frac{1}{3}}-2\ln(x^{\frac{1}{6}}+1)+2\sqrt{3}\arctan\frac{2x^{\frac{1}{6}}-1}{\sqrt{3}}$

$\displaystyle+\ln\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)+C$

Integration by parts

Product Rule:

$$\frac{d}{dx}(uv) = \frac{du}{dx}v + u\frac{dv}{dx}$$

$$d(uv) = (du)v + u(dv)$$

$$u\,dv = d(uv) - v\,du$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

eg $\int xe^x\,dx$

$$\left(\frac{d}{dx}e^x = e^x \Rightarrow de^x = e^x dx\right)$$

$$= \int x\,de^x$$

$$= xe^x - \int e^x dx \qquad u = x,\ v = e^x$$

$$= xe^x - e^x + C$$

eg $\int x\sin x\,dx$

$$\int x\sin x\,dx = \frac{1}{2}\int \sin x\,dx^2$$

deg 1

deg increases ∴ BAD

Integrate $x \to \frac{1}{2}x^2$

$$= \frac{1}{2}\left(x^2\sin x - \int x^2\,d\sin x\right)$$

Differentiate $\sin x \to \cos x$

$$= \frac{1}{2}\left(x^2\sin x - \int x^2\cos x\,dx\right)$$

deg 2 ⇒ Even worse!

Try again:

$$\int x\sin x\,dx = -\int x\,d\cos x$$

no more x (deg 0) ⇒ Easier

$$= -\left(x\cos x - \int \cos x\,dx\right)$$

Integrate $\sin x \to \cos x$

$$= -x\cos x + \int \cos x\,dx$$

Differentiate $x \to 1$

$$= -x\cos x + \sin x + C$$

deg decreases ∴ GOOD

$$\boxed{\text{The choices of } u \text{ \& } v \text{ are important.}}$$

eg $\int \arccos x \, dx$ $\quad \begin{pmatrix} u = \arccos x \\ v = x \end{pmatrix}$

$$= (\arccos x)(x) - \int x \, d(\arccos x)$$

$$= x \arccos x + \int \frac{x}{\sqrt{1-x^2}} \, dx$$

$$= x \arccos x + \frac{1}{2} \int \frac{d(x^2)}{\sqrt{1-x^2}}$$

$$= x \arccos x - \frac{1}{2} \int (1-x^2)^{-\frac{1}{2}} \, d(1-x^2)$$

$$= x \arccos x - \frac{1}{2}(2)(1-x^2)^{\frac{1}{2}} + C$$

$$= x \arccos x - \sqrt{1-x^2} + C$$

eg $\int \ln x \, dx = x \ln x - \int x \, d(\ln x)$

$$= x \ln x - \int x \cdot \frac{1}{x} \, dx$$

$$= x \ln x - x + C$$

Guideline for integration by parts on

$$\int x^n f(x) \, dx$$

- If $f(x) = \sin x, \cos x, e^x$

  then integrate $f(x)$, differentiate x^n

- If $f(x) = \arcsin x, \arccos x, \ln x$

  then integrate x^n, differentiate $f(x)$

eg

$$\int x^n \ln x \, dx = \frac{1}{n+1} \int \ln x \, d(x^{n+1})$$

$$= \frac{1}{n+1} \left[(\ln x)(x^{n+1}) - \int x^{n+1} \, d(\ln x) \right]$$

$$= \frac{x^{n+1} \ln x}{n+1} - \frac{1}{n+1} \int x^{n+1} \cdot \frac{1}{x} \, dx$$

$$= \frac{x^{n+1} \ln x}{n+1} - \frac{x^{n+1}}{(n+1)^2} + C$$

Repeated use of integration by parts

eg 1

$$\int x^2 \sin x \, dx$$

$$= -\int x^2 \, d\cos x$$

$$= -\left(x^2 \cos x - \int \cos x \, d(x^2) \right)$$

$$= -x^2 \cos x + \int 2x \cos x \, dx$$

$$= -x^2 \cos x + \int 2x \, d\sin x$$

$$= -x^2 \cos x + 2x \sin x - \int \sin x \, d(2x)$$

$$= -x^2 \cos x + 2x \sin x + 2\cos x + C$$

eg 2

$$\int e^x \cos 2x \, dx$$

integrate e^x

$$= \int \cos 2x \, de^x$$

$$= e^x \cos 2x - \int e^x \, d\cos 2x$$

$$= e^x \cos 2x + \int e^x (2 \sin 2x) \, dx$$

integrate e^x

$$= e^x \cos 2x + 2 \int \sin 2x \, de^x$$

$$= e^x \cos 2x + 2\left(e^x \sin 2x - \int e^x \, d\sin 2x \right)$$

$$= e^x \cos 2x + 2 e^x \sin 2x - 4 \underbrace{\int e^x \cos 2x \, dx}_{\text{Same as beginning}}$$

Integrate/Differentiate

$e^x \to e^x$

$\cos \to \pm \sin \to \pm \cos$

Q Which one?

A Try either one

Trick: Move that term to the left!

$$5 \int e^x \cos 2x \, dx = e^x \cos 2x + 2 e^x \sin 2x + C'$$

$$\Rightarrow \int e^x \cos 2x \, dx = \frac{1}{5}\left(e^x \cos 2x + 2 e^x \sin 2x \right) + C$$

Rmk In eg 2, we did by parts by integrating e^x twice.

Also OK to integrate trig functions twice

$$\int e^x \cos 2x \, dx$$

integrate $\cos 2x$

$$= \frac{1}{2}\int e^x \, d\sin 2x$$

$$= \frac{1}{2}\left(e^x \sin 2x - \int \sin 2x \, de^x\right)$$

$$= \frac{1}{2}\left(e^x \sin 2x - \int e^x \sin 2x \, dx\right)$$

integrate $\sin 2x$

$$= \frac{1}{2}\left(e^x \sin 2x + \frac{1}{2}\int e^x \, d\cos 2x\right)$$

$$= \frac{1}{2} e^x \sin 2x + \frac{1}{4}\left(e^x \cos 2x - \int \cos 2x \, de^x\right)$$

$$= \frac{1}{2} e^x \sin 2x + \frac{1}{4} e^x \cos 2x - \frac{1}{4}\underbrace{\int e^x \cos 2x \, dx}_{\text{same as beginning}}$$

$$\therefore \frac{5}{4}\int e^x \cos 2x \, dx = \frac{1}{2} e^x \sin 2x + \frac{1}{4} e^x \cos 2x + C'$$

$$\int e^x \cos 2x \, dx = \frac{2}{5} e^x \sin 2x + \frac{1}{5} e^x \cos 2x + C$$

However, do not integrate e^x once and trig function once.

$\boxed{\text{BAD}}$

$$\int e^x \cos 2x \, dx$$

integrate $\cos 2x$

$$= \frac{1}{2}\int e^x \, d\sin 2x$$

$$= \frac{1}{2}\left(e^x \sin 2x - \int \sin 2x \, de^x\right)$$

$$= \frac{1}{2}\left(e^x \sin 2x - \int e^x \sin 2x \, dx\right)$$

integrate e^x

$$= \frac{1}{2}\left(e^x \sin 2x - \int \sin 2x \, de^x\right)$$

$$= \frac{1}{2}\left(e^x \sin 2x - e^x \sin 2x + \int e^x \, d\sin x\right)$$

$$= \int e^x \cos 2x$$

Completely same as beginning...

Indeed line 3 is also same as line 5

line 2 is also same as line 6